MOMENT METHOD FOR THE EVALUATIONS OF HEAT TRANSFER COEFFICIENTS IN A PACKED BED[†]

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(Received 12 July 1982)

Abstract—The film heat transfer coefficient and thermal conductivity of solid particles in a packed bed can be determined from the moments of a response curve obtained by introducing a temperature pulse. The moment expressions are directly obtained from the Laplace transform of fluid temperature. The method and procedure to evaluate these two heat transport properties are described.

REFERENCES

- B(s) defined in equation (25)
- $B^*(s)$ defined in equation (25)
- $Bi = h_{\rm p} r_{\rm s} / k_{\rm p}$
- $Bi_{w} h_{w}y_{s}/k_{w}$
- $C_{\rm p}$ specific heat capacity
- $E = 3(1-\varepsilon)/\varepsilon$
- $E^* = 2y_s/R\varepsilon$
- F flow rate of fluid
- $h_{\rm p}$ film heat transfer coefficient of fluid to particle
- $h_{\rm w}$ film heat transfer coefficient of fluid to wall
- $H_{\rm pf} \ (\rho C_{\rm p})_{\rm p}/(\rho C_{\rm p})_{\rm f}$
- $H_{\rm wf} (\rho C_{\rm p})_{\rm w}/(\rho C_{\rm p})_{\rm f}$
- k thermal conductivity
- $k_{\rm s}$ effective thermal conductivity
- L length of bed
- m_n moment of *n*th order
- Q heat flux
- r_s radius of pellet
- R radius of bed
- s Laplace variable
- t time
- T temperature
- U velocity of fluid
- y transport direction in the wall
- y_s thickness of wall

Greek symbols

- ε mean voidage of bed
- $\zeta z/L$
- $\theta \quad \text{dimensionless temperature, } (T T_0)/(T_r T_0), \\ \theta_p = (T_p T_0)/(T_r T_0), \\ \theta_w = (T_w T_0)/(T_r T_0)/(T_r T_0), \\ \theta_w = (T_w T_0)/(T_r T_0)/(T_r T_0), \\ \theta_w = (T_w T_0)/(T_r T_0$
- μ_n absolute moment defined in equation (29)
- μ'_2 central absolute moment defined in equation (30)
- $\xi r/r_s$
- ρ density
- $\bar{\tau}$ dimensionless mean residence time
- τ dimensionless time, Ut/L

- $\Delta \tau$ injection time of pulse
- $\Phi_{\rm p} = (r_{\rm s}^2 U(\rho C_{\rm p})_{\rm p}/Lk_{\rm p})^{1/2}$
- $\Phi_{\rm w} = (y_{\rm s}^2 U(\rho C_{\rm p})_{\rm w}/Lk_{\rm w})^{1/2}$
- $\psi y/y_s$
- Subscripts
 - f fluid
 - 0 inlet condition
 - p solid particle
 - r reference condition, rectangular pulse input
 - s step input
 - w wall of packed bed
 - δ pulse input

1. INTRODUCTION

THE FILM heat transfer coefficient in a packed bed has been studied extensively, and Balakrishnam and Pei [1] have made a critical review of the previous investigations. The experimental work on this subject may be classified into two categories, namely, steady and unsteady-state methods. In one of the steady-state studies [2], the packed bed was heated by microwave to eliminate temperature gradient and particle-to-particle conduction, and the film heat transfer coefficient is determined by measuring the inlet and outlet temperatures of the fluid. Heat can also be generated in the pellets by means of a high frequency induction coil [3], infrared lamp [4] or electric heater [5]. Kunii and Smith [6] used a packed bed apparatus with two plates of different but constant temperatures at each end of the packed bed. Among numerous studies using the steady-state method, only Balakrishnam and Pei [2] eliminated the effect of particle-to-particle conduction, and obtained the most accurate heat transfer coefficient consequently. However, they used a very elegant experimental apparatus including a microwave generator which can not be set up easily in most laboratories.

The input-response technique was usually employed in the unsteady-state methods. The input signals include pulse [7], step [8], sinusoidal [9], cyclic variation [10, 11]. In the excellent study by Wakao [7], the heat transfer coefficient is evaluated by an optimization search method which minimizes the sum of the squares

[†] This paper was presented at the Symposium on Transport Phenomena and Applications, 1982 at Taipei, Taiwan.

of the differences between the experimental and calculated temperatures of exit fluid. Because the heat transfer coefficient is contained in one of the boundary conditions of the energy balance equation, it may not be expressed explicitly in terms of the exit temperature and operating condition. Therefore, the computation is relatively tedious and time consuming.

Usually the thermal conductivity of the material of solid particles is taken as the thermal conductivity of solid particles. In case that value is not suitable (e.g. for porous materials) or is not available, the thermal conductivity can be evaluated by the methods suggested by Butt [12], Masamune and Smith [13], and Woodside and Messmer [14].

The method of moments has been applied for measuring adsorption rate constant [15], surface, diffusivity [16], and effective diffusivity [17] in catalyst pellets and diffusivity in a single catalyst bed [18, 19] and for evaluating kinetic parameters in fixed bed [20], slurry [21] and trickle-bed reactors [22–24]. This method provides a simple way of analyzing a system or evaluating some parameters without expressing the concentration in terms of space and time. The expressions of moments can be derived from the Laplace transform of concentration. In this work, the method of moment analysis is employed to obtain the moment expressions for the response curve from a packed bed to which a temperature signal is introduced. Energy balance equations are written for this unsteadystate system and are transformed to ordinary differential equations by applying the Laplace transformation. After these differential equations are solved to obtain the fluid temperature in the Laplace domain, the moment expressions are directly obtained by differentiating the Laplace transform of fluid temperature and setting the Laplace variable to zero.

2. MODEL FOR HEAT TRANSFER IN A PACKED BED

For the purpose of evaluating the film heat transfer coefficient and thermal conductivity of solid particles, let us consider a steady-state adiabatic packed bed through which a fluid of constant temperature passes at a constant flow rate. When a pulse of fluid of different temperature (temperature pulse) is introduced at the entrance of the packed bed, this temperature wave will be attenuated along the axial direction. Due to the effects of dispersion, heat transfer across the fluid film on the solid particles and conduction within the particles, the temperature wave will be retained in the bed longer than the mean residence time of fluid, and its shape will be distorted. By using the plug flow model and assuming constant thermal properties, we can write the following energy conservation equation for the packed bed:

$$\frac{\partial T}{\partial t} = -U \frac{\partial T}{\partial z} - \frac{3(1-\varepsilon)}{\varepsilon r_{s}} \frac{k_{p}}{(\rho C_{p})_{f}} \times \frac{\partial T_{p}}{\partial r} \bigg|_{r=r_{s}} - \frac{2k_{w}}{\varepsilon R(\rho C_{p})_{f}} \frac{\partial T_{w}}{\partial y} \bigg|_{y=y_{s}}.$$
 (1)

An energy balance for solid particles gives

$$\frac{\partial T_{\mathbf{p}}}{\partial t} = \frac{k_{\mathbf{p}}}{(\rho C_{\mathbf{p}})_{\mathbf{p}}} \left(\frac{\partial^2 T_{\mathbf{p}}}{\partial r^2} + \frac{2}{r} \frac{\partial T_{\mathbf{p}}}{\partial r} \right). \tag{2}$$

Because the wall of the bed will absorb and release part of the energy, we should also consider the heat transfer in the wall. Due to the fact that the wall thickness is usually far less than the diameter of the bed, we can consider the wall as a flat plate of thickness y_s (Fig. 1). Thus the energy balance gives

$$\frac{\partial T_{\mathbf{w}}}{\partial t} = \frac{k_{\mathbf{w}}}{(\rho C_{\mathbf{v}})_{\mathbf{w}}} \frac{\partial^2 T_{\mathbf{w}}}{\partial y^2}.$$
(3)

These three partial differential equations are accompanied by the following initial and boundary conditions:

At
$$t = 0$$

 $T = T_0$ for $0 \le z \le L$
 $T_p = T_0$ $0 \le r \le r_s$ (4)
 $T_w = T_0$ $0 \le y \le y_s$.
For $t > 0$
 $T = T_r \delta(t)$ at $z = 0$, (5)

$$\frac{\partial T_{\mathbf{p}}}{\partial r} = 0 \qquad \text{at } r = 0, \tag{6}$$

$$k_{\rm p} \frac{\partial T_{\rm p}}{\partial r} = h_{\rm p} (T - T_{\rm p})$$
 at $r = r_{\rm s}$,

$$\frac{\partial T_{\mathbf{w}}}{\partial y} = 0 \qquad \text{at } y = 0,$$

$$\frac{\partial T_{\mathbf{w}}}{\partial y} = 0 \qquad (7)$$

$$k_{\mathbf{w}} \frac{\partial \mathbf{I}_{\mathbf{w}}}{\partial y} = h_{\mathbf{w}}(T - T_{\mathbf{w}})$$
 at $y = y_{\mathbf{s}}$.

It is convenient to write the above equations and the corresponding initial and boundary conditions in dimensionless forms

$$\frac{\partial \theta}{\partial \tau} = -\frac{\partial \theta}{\partial \zeta} - \frac{EH_{\rm pf}}{\Phi_{\rm p}^2} \frac{\partial \theta_{\rm p}}{\partial \xi} \bigg|_{\xi=1} - \frac{E^*H_{\rm wf}}{\Phi_{\rm w}^2} \frac{\partial \theta_{\rm w}}{\partial \psi} \bigg|_{\psi=1}, \quad (8)$$



FIG. 1. Schematic diagram of a packed bed.

$$\Phi_{\rm p}^2 \frac{\partial \partial_{\rm p}}{\partial \tau} = \frac{\partial^2 \partial_{\rm p}}{\partial \xi^2} + \frac{2}{\xi} \frac{\partial \partial_{\rm p}}{\partial \xi},\tag{9}$$

$$\Phi_{\mathbf{w}}^{2} \frac{\partial \theta_{\mathbf{w}}}{\partial \tau} = \frac{\partial^{2} \theta_{\mathbf{w}}}{\partial \psi^{2}},\tag{10}$$

at
$$\tau = 0$$
;

$$\theta = 0$$
 for $0 \le \zeta \le 1$,
 $\theta_{p} = 0$ $0 \le \xi \le 1$, (11)

$$\theta_{\rm w} = 0 \qquad 0 \leqslant \psi \leqslant 1,$$

for $\tau \ge 0$:

$$\theta = \delta(t)$$
 at $\zeta = 0$, (12)

$$\frac{\partial \theta_{\mathbf{p}}}{\partial \xi} = 0 \qquad \text{at } \xi = 0, \tag{13}$$

$$\frac{\partial \theta_{\mathbf{p}}}{\partial \xi} = Bi(\theta - \theta_{\mathbf{p}}) \quad \text{at } \xi = 1,$$

$$\frac{\partial \theta_{\mathbf{w}}}{\partial \psi} = 0 \qquad \text{at } \psi = 0,$$

$$\frac{\partial \theta_{\mathbf{w}}}{\partial \psi} = Bi_{\mathbf{w}}(\theta - \theta_{\mathbf{w}}) \text{ at } \psi = 1.$$
(14)

The Laplace transforms of equations (8)–(10) and juations (12)–(14) with respect to τ are

$$\bar{\theta} = -\frac{\mathrm{d}\bar{\theta}}{\mathrm{d}\zeta} - \frac{EH_{\mathrm{pf}}}{\Phi_{\mathrm{p}}^{2}} \frac{\mathrm{d}\bar{\theta}_{\mathrm{p}}}{\mathrm{d}\xi} \bigg|_{\xi=1} - \frac{E^{*}H_{\mathrm{wf}}}{\Phi_{\mathrm{w}}^{2}} \frac{\mathrm{d}\bar{\theta}_{\mathrm{w}}}{\mathrm{d}\psi} \bigg|_{\psi=1}$$
(15)

$$s\Phi_{\rm p}^2\overline{\partial}_{\rm p} = \frac{{\rm d}^2\overline{\partial}_{\rm p}}{{\rm d}\xi^2} + \frac{2}{\xi}\frac{{\rm d}\overline{\partial}_{\rm p}}{{\rm d}\xi},\qquad(16)$$

$$s\Phi_{\rm w}^2\bar{\theta}_{\rm w} = \frac{{\rm d}^2\bar{\theta}_{\rm w}}{{\rm d}\psi^2},\tag{17}$$

$$\overline{\theta} = 1,$$
 at $\zeta = 0,$ (18)

$$\frac{\mathrm{d}\theta_{\mathrm{p}}}{\mathrm{d}\xi} = 0, \qquad \text{at } \xi = 0, \tag{19}$$

$$\frac{d\theta_{p}}{d\xi} = Bi(\bar{\theta} - \bar{\theta}_{p}), \quad \text{at } \xi = 1,$$

$$\frac{d\bar{\theta}_{w}}{d\psi} = 0, \quad \text{at } \psi = 0,$$
(20)

$$\frac{\mathrm{d}\overline{\theta}_{\mathrm{w}}}{\mathrm{d}\psi} = Bi_{\mathrm{w}}(\overline{\theta} - \overline{\theta}_{\mathrm{w}}), \text{ at } \psi = 1.$$

ne solutions of equations (16) and (17) are

$$= Bi\overline{\theta} \sinh \left(\xi \Phi_{p} s^{1/2}\right)$$

+ $\left[\xi \Phi_{p} s^{1/2} \cosh(\Phi_{p} s^{1/2}) + \xi(Bi-1) \sinh \left(\Phi_{p} s^{1/2}\right)\right]$ (21)

d

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$$= Bi_{w}\overline{\theta} \cosh(\psi \Phi_{w} s^{1/2})
\div [\Phi_{w} s^{1/2} \sinh(\Phi_{w} s^{1/2}) + Bi_{w} \cosh(\Phi_{w} s^{1/2})]. \quad (22)$$

From these two expressions, it is easy to show that

$$\left. \frac{\mathrm{d}\bar{\partial}_{\mathbf{p}}}{\mathrm{d}\xi} \right|_{\xi=1} = B(s)\bar{\partial} \tag{23}$$

and

$$\left. \frac{\mathrm{d}\overline{\partial}_{\mathbf{w}}}{\mathrm{d}\psi} \right|_{\psi=1} = B^*(s)\overline{\partial} \tag{24}$$

where

$$B(s) = Bi [\Phi_{p} s^{1/2} \cosh(\Phi_{p} s^{1/2}) - \sinh(\Phi_{p} s^{1/2})]$$

+ $[\Phi_{p} s^{1/2} \cosh(\Phi_{p} s^{1/2}) + (Bi - 1) \sinh(\Phi_{p} s^{1/2})]$

and

$$B^{*}(s) = Bi_{w} \Phi_{w} s^{1/2} \sinh(\Phi_{w} s^{1/2})$$

$$\div [\Phi_{w} s^{1/2} \sinh(\Phi_{w} s^{1/2}) + Bi_{w} \cosh(\Phi_{w} s^{1/2})]. \quad (25)$$

The substitution of equations (23) and (24) into equation (15) and the use of boundary condition (18) gives

$$\overline{\theta} = \exp\left\{-\left[s + \frac{EH_{\rm pf}}{\Phi_{\rm p}^2}B(s) + \frac{E^*H_{\rm wf}}{\Phi_{\rm w}^2}B^*(s)\right]\zeta\right\}.$$
 (26)

3. EXPRESSIONS FOR MOMENTS

The moments of dimensionless temperature at the exit of a packed bed are related to the exit temperature in the Laplace domain by

$$m_n = (-1)^n \lim_{s \to 0} \frac{\mathrm{d}^n}{\mathrm{d}s^n} \,\overline{\theta}(s,1) \tag{27}$$

where the moments are defined as

$$m_n = \int_0^\infty \tau^n \theta(\tau, 1) \,\mathrm{d}\tau. \tag{28}$$

The absolute moments are

$$\mu_n = \frac{m_n}{m_0} \tag{29}$$

and the second central absolute moment is

$$\mu_2' = \frac{1}{m_0} \int_0^\infty (\tau - \bar{\tau})^2 \theta(\tau, 1) \,\mathrm{d}\tau \tag{30}$$

where $\bar{\tau}$ is the mean residence time of the effluent temperature wave and is equal to μ_1 .

3.1. Pulse input

By substituting equation (26) into the above equations, we can obtain

$$m_0 = 1,$$
 (31)

$$\mu_1 = m_1 = 1 + \frac{1}{3}EH_{\rm pf} + E^*H_{\rm wf},\tag{32}$$

$$\mu'_{2} = \mu_{2} - \mu_{1}^{2} = \frac{2}{9} E H_{pf} \Phi_{p}^{2} \left(0.2 + \frac{1}{Bi} \right) + E^{*} H_{wf} \Phi_{w}^{2} \left(\frac{2}{3} + \frac{2}{Bi_{w}} \right)$$
(33)

or

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$$\frac{\mu'_2}{EH_{\rm pf}} = \frac{2}{9} \Phi_{\rm p}^2 \left(0.2 + \frac{1}{Bi} \right) + \frac{E^* H_{\rm pf}}{EH_{\rm wf}} \Phi_{\rm w}^2 \left(\frac{2}{3} + \frac{2}{Bi_{\rm w}} \right).$$
(34)

Note that the zero moment is unity. This means that all the energy of the pulse input is carried to the exit, because no heat is lost or generated in the adiabatic packed bed.

3.2. Rectangular pulse input

If a rectangular temperature pulse is introduced instead of a delta function, the boundary condition (5) will be replaced by

$$T = \begin{cases} T_{\mathbf{r}} & 0 \le t \le \Delta t \\ T_0 & \text{for } \Delta t < t \end{cases} \quad \text{at } z = 0 \qquad (35)$$

or in dimensionless form by

$$\theta = \begin{cases} 1 & \text{for } 0 \leq \tau \leq \Delta \tau \\ 0 & \text{for } \Delta \tau < \tau \end{cases} \quad \text{at } \zeta = 0.$$
 (36)

The use of this boundary condition for equation (15) results in

$$\overline{\theta} = \frac{1}{s} \left[1 - \exp\left(-s\Delta\tau\right) \right]$$
$$\exp\left\{ -\left[s + \frac{EH_{\text{pf}}}{\Phi_{\text{p}}^2} B(s) + \frac{E^*H_{\text{wf}}}{\Phi_{\text{w}}^2} B^*(s) \right] \zeta \right\} \quad (37)$$

and

$$(m_0)_r = \Delta \tau, \tag{38}$$

$$(\mu_1)_r = \mu_1 + \Delta \tau, \tag{39}$$

$$(\mu'_2)_r = \mu_2 - \frac{2}{3} (\Delta \tau)^2.$$
 (40)

Note that equation (37) reduces to equation (26) when $\Delta \tau$ approaches zero.

3.3. Step input

Because a pulse input often results in an uncertain tail in the response curve, and hence gives unusually large moments, a step input, which avoids the emphasis to the uncertain tail, is often used. However, the response curve of a step input, which is called a breakthrough curve, may not be used to obtain the moments of the response curve, because the measured moments will be infinite. Instead, the response curve of a step input can be applied to evaluate the moments of the response curve of a pulse input by using the following relation:

$$\frac{(T-T_0)_{\delta}}{\int_0^{\infty} (T-T_0)_{\delta} dt} = \frac{d}{dt} \left[\frac{(T-T_0)_s}{(T_{\infty}-T_0)_s} \right]$$
(41)

where the subscripts δ and s refer to the response curves of pulse input and step input, and $T_{\infty} = T(L, t)$ at $t = \infty$. After being written as a dimensionless form by taking $T_r = T_{\infty}$, the above equation is further transformed to

where

$$m_0 = \int_0^\infty \theta_\delta(1,\tau) \,\mathrm{d}\tau. \tag{43}$$

(42)

Hence,

$$\mu_1 = \frac{m_1}{m_0} = \int_0^\infty \left[1 - \theta_s(1, \tau) \right] d\tau, \tag{44}$$

$$\mu_{2} = \int_{0}^{\infty} \left[\frac{1}{2} - 2\tau \theta_{s}(1,\tau) \right] d\tau.$$
 (45)

It should be mentioned that μ_1 and μ_2 are the moments for a pulse input evaluated from the response curve of a step input.

 $\overline{\theta}_{\delta}(1,s) = m_0 s \overline{\theta}_s(1,s)$

4. EVALUATION OF HEAT TRANSFER COEFFICIENTS

In the case when the packed bed has a large diameterto-wall thickness ratio so that the heat capacity of the wall is relatively small, the terms containing E^*H_{wf} can be neglected in equations (32) and (33). Thus,

$$\mu_1 = m_1 = 1 + \frac{1}{3}EH_{\rm pf},\tag{46}$$

$$\frac{\mu'_2}{EH_{\rm pf}} = \frac{2}{9} \Phi_{\rm p}^2 \left(0.2 + \frac{1}{Bi} \right). \tag{47}$$

It is seen that the first moment depends only on the value EH_{pf} which relates to the void fraction of the bed and the specific heat capacities of solid particles and fluid. Since the void fraction can be easily measured, the value of EH_{pf} can be evaluated without measuring the response curve. However, equation (46) can be applied for checking the accuracy of the value of the first moment calculated from the response signal.

After the value of EH_{pf} is found, μ'_2 is a function of Φ_p and *Bi*. It is seen that these two parametric groups contain two heat transfer coefficients, k_p and h_p . Usually, k_p can be determined separately as has been mentioned in Section 1. Thus, the value of h_p can be determined from equation (48) which is rearranged from equation (47) after μ'_2 is calculated from the response curve,

$$Bi = \left[\frac{9\mu'_2}{2EH_{\rm pf}\Phi_{\rm p}^2} - 0.2\right]^{-1}.$$
 (48)

If k_p cannot be measured or found in advance, its value can be determined with h_p simultaneously. In such a case, at least two experimental values of μ'_2 are needed. These two values should be evaluated under the restriction that h_p is kept unchanged. The best way to have different values of μ'_2 while keeping the same value of h_p is to change the length of the packed bed.

Figure 2 shows a plot of μ'_2/EH_{pf} vs ϕ_p with the Biot number as a parameter. It is obvious that a large *Bi* or Φ_p gives a large μ'_2 . This is because that a large h_p and/or a small k_p gives a high heat transfer rate across the film.



FIG. 2. Effects of Biot number and Thiele modulus of particles on μ'_2/EH_{pf} .

Because the wall effect decreases as the ratio of wall thickness to bed diameter decreases, the effect will be negligible when the ratio approaches zero. Therefore, we can obtain more precise values for the heat transfer coefficients by extrapolating the curve of $h_p - y_s/R$ and $k_p - y_s/R$ to the point of $y_s/R = 0$.

The effect of y_s/R on μ'_2 is illustrated in Fig. 3. This was plotted by applying equation (34). The dimensionless group E^*H_{pt}/EH_{wf} represents y_s/R . The line $\Phi_w = 2(=\Phi_p)$ shows the case where the packings and the wall of the packed bed have the same heat capacity and same thermal conductivity. It is seen that the effect of y_s/R is quite large. However, if Φ_w is small (Φ_p/Φ_w)



FIG. 3. Effect of the ratio of wall thickness to radius of packed bed on μ'_2 .

large) the effect of y_s/R is quite small and can be neglected. The uses of metallic wall and nonmetallic beads fall into this category. Under such a case, the wall effect can be neglected and the uses of equations (44) and (45) for evaluating h_p will give reasonable results.

After the values or k_p and h_p have been determined, those of k_w and h_w can also be evaluated from equation (33) by the same method for h_p and k_p .

5. PROPOSED EXPERIMENTAL APPARATUS AND OPERATING METHODS

In this section an experimental apparatus and its operating method are proposed for the evaluation of heat transfer coefficients.

Figure 4(a) shows the proposed experimental apparatus. The packed bed is made of two coaxial tubes. In order to minimize the heat transfer across the tubes, the inner surface of the outer tube is polished or silver-coated and the air in the annular space is evacuated. A four-way valve is fitted close to the entrance of the packed bed. At the start of the experiment, fluid of temperature T_0 passes continuously through the packed bed via two ports of the fourway valve, while another fluid of temperature T_r flows through the other two ports to the vent. After reaching a steady state, the fluid of temperature T_r is introduced to the packed bed by turning the four-way valve. If the valve is not turned back to the original position, a step input is obtained. The input is rectangular in shape if the valve is turned back. When these two consecutive turnings are within a very short time interval, the input can be considered as a delta function.

A set-up as shown in Fig. 4(b) can also be used for the experimental work. In this set-up, a microwave generator is used to heat up the fluid. After the microwave source is turned on, the fluid temperature at the entrance of the packed bed will increase and reach a steady value. Because the time to reach the steady



FIG. 4. Sketch of the proposed experimental apparatus.

temperature can be made very short, the temperature change at the inlet of the packed bed may be approximated as a step change. If the microwave is not turned off an approximate step input is obtained. If the microwave is turned off after a period of time, an approximately rectangular input results.

It should be pointed out that a pulse input is not suitable for the evaluation of the heat transfer coefficients. This is not only due to the uncertain tail, as was mentioned above, but also due to the fact that it is difficult to detect the temperature response accurately because the input energy is relatively small.

6. CONCLUDING REMARKS

This work presents a method of moment analysis for the evaluation of heat transfer coefficients in a packed bed by using a plug flow model. When axial heat transfer may not be neglected, the 1-dim. dispersion model or 1-dim. conduction-dispersion model should be applied. For such a case, the method and procedure for evaluating the moment expressions and determining the transfer coefficients are the same as those shown in this work.

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METHODE DU MOMENT POUR EVALUER LES COEFFICIENTS DE TRANSFERT THERMIQUE DANS UN LIT FIXE

Résumé—Le coefficient de transfert thermique de film et la conductivité thermique de particules solides dans un lit fixe peuvent être déterminés par les moments d'une courbe de réponse obtenue en introduisant une impulsion de température. Les expressions de moment sont directement obtenues par la transformée de Laplace de la température du fluide. On décrit la méthode et la procédure pour évaluer ces deux propriétés de transport thermique.

MOMENTENMETHODE FÜR DIE BERECHNUNG VON WÄRMEÜBERGANGSKOEFFIZIENTEN IN EINEM FESTBETT

Zusammenfassung-Der Wärmeübergangskoeffizient und die Wärmeleitfähigkeit der Partikel in einem Festbett können mit Hilfe der Momente der Übergangsfunktion, die man als Antwort auf einen Temperaturimpuls erhält, bestimmt werden. Die Momentenausdrücke werden direkt aus der Laplacetransformierten Fluidtemperatur erhalten. Die Methode und das Verfahren zur Bestimmung dieser beiden Transportgrößen werden beschrieben.

МЕТОД МОМЕНТОВ ДЛЯ РАСЧЕТА КОЭФФИЦИЕНТОВ ТЕПЛООБМЕНА В ПЛОТНОМ СЛОЕ

Аннотация—Коэффициент теплоотдачи к пленке и коэффициент теплопроводности твердых частиц в плотном слое могут быть определены с помощью моментов кривой отклика на температурный импульс. Выражения для момента получаются непосредственно из преобразования Лапласа для температуры жидкости. Описывается метод и способ расчета двух указанных характеристик переноса тепла.